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Vector boson decays of the Higgs boson

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Abstract. We derive the width of the Higgs boson into vector bosons. General formulas are derived both for the on–shell decay $H \to VV$ as well for the off–shell decays, $H \to V^*V$ and $H \to V^*V^*$, where $V = \gamma, W^{\pm}, Z^0$. For the off-shell decays the width of the decaying vector boson is properly included. The formulas are valid both for the Standard Model as well as for arbitrary extensions. As an example we study in detail the gauge-invariant effective Lagrangian models where we can have sizable enhancements over the Standard Model that could be observed at LEP.

1 Introduction

In recent years it has been established [1] with great precision (in some cases better than 0.1%) that the interactions of the gauge bosons with the fermions are described by the Standard Model (SM) [2]. However other sectors of the SM have been tested to a much lesser degree. In fact only now we are beginning to probe the self–interactions of the gauge bosons through their pair production at the Tevatron [3] and LEP [4] and the Higgs sector, responsible for the symmetry breaking has not yet been tested.

A more complicated symmetry breaking sector can introduce modifications in the couplings of the Higgs boson with the vector bosons. It is therefore important to have expressions for the decay widths of the Higgs boson into vector bosons that are valid for an arbitrary extension of the SM. For the region of the Higgs boson mass relevant for searches at LEP II and LHC it is necessary that the vector bosons in the decays can be off–shell.

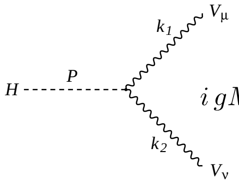
In this paper we derive the complete set of formulas for the decay widths of the Higgs boson in vector bosons. The formulas are valid both for the Standard Model (SM) and for any arbitrary extension. For the case of the decay into the $W^{\pm}$ and Z^0 the formulas are also valid for off–shell decays. This is important for Higgs boson masses close to the threshold of the production of one or two real vector bosons. Many of these results have appeared before in the literature [5–10], sometimes for particular cases, but we think that it will be very useful for the Higgs boson search at LEP and at LHC to have the general results in a consistent notation.

The paper is organized as follows. In Sect. 2 the decays $H \to VV$ where $V = W^{\pm}, Z^0$ are calculated. The decays $H \to \gamma\gamma$ and $H \to \gamma Z^0$, that in the SM proceed at one–loop level, are reviewed in Sects. 3 and 4, respectively. In Sect. 5 the off-shell 3–point functions $Z^* \to H\gamma$ and $\gamma^* \to H\gamma$ are given in a consistent notation both for the SM as well as for any of its extensions. In Sect. 6 we give an example of physics Beyond de Standard Model (BSM) and in Sect. 7 a brief discussion of our results and a comparison with previous ones is presented.

2 The decays $H \to VV$

2.1 The HVV couplings

We consider the most general couplings of the Higgs H with the $W^{\pm}$ and Z^0. These are



$$i\, g M_V \left(g_{\mu\nu} + T^V_{\mu\nu}\right) \tag{1}$$

where $V = W, Z$ and $T^W_{\mu\nu}$ and $T^Z_{\mu\nu}$ are the extra contributions from new physics Beyond the Standard Model (BSM). In general they will depend on the momenta P, k_1 and k_2, but as we will see, we will not need their exact expressions to get the final formulas.

2.2 The on–shell decay $H \to VV$

We now consider the on–shell decay $H \to VV$. To be precise we derive the expression for $H \to W^+W^-$ and then present a final result valid also for $H \to Z^0Z^0$. We consider the kinematics given in Fig. 1.
The differential width is given by [11]

$$d\Gamma = \frac{1}{32\pi^2} \sum_{pol} |\mathcal{M}|^2 \frac{|\mathbf{k}_1|}{M_H^2} d\Omega_{\mathbf{k}_1} \tag{2}$$

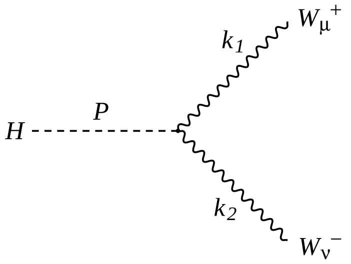


Fig. 1.

where

$$\mathcal{M} = i\, gM_W \epsilon^\mu(k_1)\epsilon^\nu(k_2)\left(g_{\mu\nu} + T^W_{\mu\nu}\right) \tag{3}$$

We get therefore

$$\begin{aligned}\sum_{pol}|\mathcal{M}|^2 &= (gM_W)^2 \left(-g^{\mu\alpha} + \frac{k_1^\mu k_1^\alpha}{M_W^2}\right)\left(-g^{\nu\beta} + \frac{k_2^\nu k_2^\beta}{M_W^2}\right)\\ &\quad \times \left(g_{\mu\nu} + T^W_{\mu\nu}\right)\left(g_{\alpha\beta} + T^W_{\alpha\beta}\right)\\ &= \Bigg[2 + \frac{(k_1\cdot k_2)^2}{M_W^4} + 2T^{W\alpha}_{\alpha} - 2\frac{k_1^\alpha k_1^\beta}{M_W^2}\\ &\quad \times T^W_{\alpha\beta} - 2\frac{k_2^\alpha k_2^\beta}{M_W^2}T^W_{\alpha\beta} + 2\frac{k_1\cdot k_2 k_1^\alpha k_2^\beta}{M_W^4}T^W_{\alpha\beta}\\ &\quad + T^W_{\mu\nu}T^{W\mu\nu} - \frac{k_{1\mu}k_1^\alpha}{M_W^2}T^{\mu\nu}T^W_{\alpha\nu} - \frac{k_{2\nu}k_2^\beta}{M_W^2}T^{\alpha\nu}T^W_{\alpha\beta}\\ &\quad + \frac{k_1^\mu k_2^\nu}{M_W^2}T^W_{\mu\nu}\frac{k_1^\alpha k_2^\beta}{M_W^2}T^W_{\alpha\beta}\Bigg]\end{aligned} \tag{4}$$

Now, using

$$\begin{aligned}k_1\cdot k_2 &= \frac{1}{2}(M_H^2 - 2M_W^2)\\ &= \frac{1}{2}\sqrt{M_H^4\lambda(M_W^2, M_W^2; M_H^2) + 4M_W^4}\end{aligned} \tag{5}$$

where

$$\lambda(x,y;z) = \left(1 - \frac{x}{z} - \frac{y}{z}\right)^2 - 4\frac{xy}{z^2} \tag{6}$$

and defining

$$\begin{aligned}X(p_1,p_2,M_H,T^V) &\equiv 4\Bigg[2\frac{p_1^2p_2^2}{M_H^4}T^{V\alpha}_{\alpha} - 2\frac{p_2^2}{M_H^2}\frac{p_1^\alpha p_1^\beta}{M_H^2}T^V_{\alpha\beta}\\ &-2\frac{p_1^2}{M_H^2}\frac{p_2^\alpha p_2^\beta}{M_H^2}T^V_{\alpha\beta} + 2\frac{p_1\cdot p_2 p_1^\alpha p_2^\beta}{M_H^4}T^V_{\alpha\beta}\\ &+\frac{p_1^2p_2^2}{M_H^4}T^V_{\mu\nu}T^{V\mu\nu} - \frac{p_2^2}{M_H^2}\frac{p_{1\mu}p_1^\alpha}{M_H^2}T^{V\mu\nu}T^V_{\alpha\nu}\\ &-\frac{p_1^2}{M_H^2}\frac{p_{2\nu}p_2^\beta}{M_H^2}T^{V\alpha\nu}T^V_{\alpha\beta} + \frac{p_1^\mu p_2^\nu}{M_H^2}T^V_{\mu\nu}\frac{p_1^\alpha p_2^\beta}{M_H^2}T^V_{\alpha\beta}\Bigg]\end{aligned} \tag{7}$$

we can write

$$\begin{aligned}\sum_{pol}|\mathcal{M}|^2 &= (gM_W)^2\frac{M_H^4}{4M_W^4}\times\Big[\lambda(M_W^2,M_W^2;M_H^2)\\ &+12\frac{M_W^4}{M_H^4} + X(k_1,k_2,M_H,T^W)\Big]\end{aligned} \tag{8}$$

It is easy to see that the 4–momenta k_1 and k_2 will only appear in the square bracket of (8) as the scalar products like $k_1\cdot k_2$, $P\cdot k_1$ and $P\cdot k_2$. These can all be written in terms of the masses and therefore there is no angular dependence in $d\Gamma$. Noticing also that

$$|\mathbf{k}_1| = \frac{1}{2}M_H\sqrt{\lambda(M_W^2,M_W^2;M_H^2)} \tag{9}$$

we can finally write

$$\begin{aligned}\Gamma &= \frac{g^2M_H^3}{64\pi M_W^2}\sqrt{\lambda(M_W^2,M_W^2;M_H^2)}\times\Big[\lambda(M_W^2,M_W^2;M_H^2)\\ &+\frac{M_W^4}{M_H^4} + X(k_1,k_2,M_H,T^W)\Big]\end{aligned} \tag{10}$$

which can be written in terms of G_F as

$$\begin{aligned}\Gamma &= \frac{G_FM_H^3}{8\pi\sqrt{2}}\sqrt{\lambda(M_W^2,M_W^2;M_H^2)}\times\Big[\lambda(M_W^2,M_W^2;M_H^2)\\ &+12\frac{M_W^4}{M_H^4} + X(k_1,k_2,M_H,T^W)\Big]\end{aligned} \tag{11}$$

Now for the decay $H\to Z^0Z^0$ everything is similar except that we have to divide by a factor of 2 because we have two identical particles in the final state. Introducing $\delta_V = 2(1)$ for $V = W(Z)$, respectively, we can write both decays in a single formula

$$\begin{aligned}\Gamma &= \delta_V\frac{G_FM_H^3}{16\pi\sqrt{2}}\sqrt{\lambda(M_V^2,M_V^2;M_H^2)}\times\Big[\lambda(M_V^2,M_V^2;M_H^2)\\ &+12\frac{M_V^4}{M_H^4} + X(k_1,k_2,M_H,T^V)\Big]\end{aligned} \tag{12}$$

where λ and X are given in (6) and (7). The SM part of (12) agrees with (5) of [10] and it is also in agreement with [9]. The term proportional to X represents the extra contributions from physics beyond the SM and is in agreement with the results of [8] as we will explain in Sect. 6.

2.3 The off–shell decay $H \to VV^*$

We now consider the off–shell decay $H\to VV^*$. To be precise we derive the expression for $H\to W^+W^{-*}\to W^+f_i\overline{f'}_i$ and then present a final result valid for all cases. We consider the kinematics given in Fig. 2, where $(f_i,\overline{f'}_i)$ represents one of the decay channels of the W^-, for instance, $(e^-,\overline{\nu}_e)$. Using the conventions of [11], we can write the differential width as

$$d\Gamma = \frac{(2\pi)^4}{2M_H}\sum_{pol}|\mathcal{M}|^2 d\Phi_3 \tag{13}$$

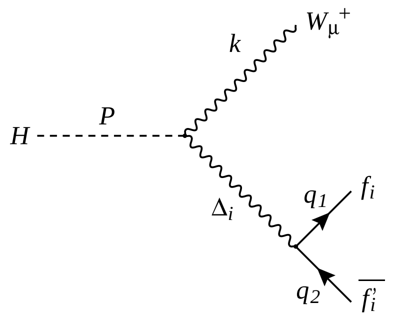


Fig. 2.

where $d\Phi_3$ is the phase space for 3 particles that we write as [11]

$$d\Phi_3(P;k,q_1,q_2) = d\Phi_2(P;k,\Delta_i)d\Phi_2(\Delta_i;q_1,q_2) \times (2\pi)^3 d\Delta_i^2 \tag{14}$$

with

$$\Delta_i = q_1 + q_2; \Delta_i^2 = (q_1+q_2)^2 \tag{15}$$

But the 2–body phase space in the rest frame of the decaying W can be written as

$$d\Phi_2(\Delta_i;q_1,q_2) = (2\pi)^{-6} \frac{|\mathbf{q^*}_1|}{4\sqrt{\Delta_i^2}}\, d\Omega_1^* = \frac{(2\pi)^{-6}}{8}\, d\Omega_1^* \tag{16}$$

where the last equality holds for massless decaying products of the W that we will assume and Ω_1^* is the solid angle of the particle with momentum q_1 in the rest frame of the decaying W. Also the 2–body phase space of the decaying H can be written as

$$d\Phi_2(P;k,\Delta_i) = \frac{(2\pi)^{-6}}{8}\sqrt{\lambda(M_W^2,\Delta_i^2;M_H^2)}\, d\Omega_{\mathbf{k}} \tag{17}$$

Putting everything together we get

$$d\Gamma = \frac{(2\pi)^{-5}}{128 M_H}\sqrt{\lambda(M_W^2,\Delta_i^2;M_H^2)} \sum_{pol} |\mathcal{M}|^2\, d\Omega_{\mathbf{k}}\, d\Delta_i^2 d\Omega_1^* \tag{18}$$

Neglecting the fermion masses the matrix element $\mathcal{M}$ is

$$\mathcal{M} = (gM_W)\epsilon^\alpha(k)\left(g_{\mu\alpha} + T^W_{\mu\alpha}\right) \times \frac{1}{D(\Delta_i^2)}\frac{g}{2\sqrt{2}}\,\overline{u}(q_1)\gamma^\mu(1-\gamma_5)\,v(q_2) \tag{19}$$

where

$$D(\Delta_i^2) = \Delta_i^2 - M_W^2 + iM_W\Gamma_W \tag{20}$$

We obtain for the matrix element squared

$$\begin{aligned}\sum_{pol}|\mathcal{M}|^2 &= (gM_W)^2\frac{1}{|D(\Delta_i^2)|^2}\left(-g^{\alpha\beta} + \frac{k^\alpha k^\beta}{M_W^2}\right)\\ &\times\left(g_{\mu\alpha}+T^W_{\mu\alpha}\right)\left(g_{\nu\beta}+T^W_{\nu\beta}\right)\\ &\times\frac{g^2}{8}\,\mathrm{Tr}\Big[\not{q}_1\gamma^\mu(1-\gamma_5)\not{q}_2\gamma^\nu(1-\gamma_5)\Big]\\ &= (gM_W)^2\frac{1}{|D(\Delta_i^2)|^2}\left(-g^{\alpha\beta}+\frac{k^\alpha k^\beta}{M_W^2}\right)\\ &\times\left(g_{\mu\alpha}+T^W_{\mu\alpha}\right)\left(g_{\nu\beta}+T^W_{\nu\beta}\right)\\ &\times\frac{48\pi\Gamma_i}{M_W}\Big[q_1^\mu q_2^\nu + q_2^\mu q_1^\nu - g^{\mu\nu} q_1\cdot q_2\Big]\end{aligned} \tag{21}$$

where $\Gamma_i = g^2/(48\pi)\,M_W$ is the decay width $W \to f_i\overline{f'}_i$. Looking at (18) and (21) we realize that the only dependence on the solid angle Ω_1^* is inside the square bracket in (21). Then the integrals we have to evaluate are of the form

$$I^{\alpha\beta} = \int d\Omega_1^* q_1^\alpha q_2^\beta \tag{22}$$

These can be easily done if we realize that in the rest frame of the decaying W the only 4–vector available is Δ_i. We should have then

$$I^{\alpha\beta} = A\Delta_i^\alpha\Delta_i^\beta + B\Delta_i^2 g^{\alpha\beta} \tag{23}$$

Multiplying the last equation respectively with $g_{\alpha\beta}$ and with $\Delta_{i\alpha}\Delta_{i\beta}$ and noticing that $\Delta_i\cdot q_1 = \Delta_i\cdot q_2 = 1/2\Delta_i^2$ we get a system of equations for A and B

$$\begin{cases} A + 4B = 2\pi \\ A + B = \pi \end{cases} \tag{24}$$

which gives $A = 2\pi/3$ and $B = \pi/3$. We get then

$$\int d\Omega_1^* q_1^\alpha q_2^\beta = \frac{\pi}{3}\left(2\Delta_i^\alpha\Delta_i^\beta + \Delta_i^2 g^{\alpha\beta}\right) \tag{25}$$

and

$$\int d\Omega_1^*\left[q_1^\mu q_2^\nu + q_2^\mu q_1^\nu - g^{\mu\nu}q_1\cdot q_2\right] = \frac{4\pi}{3}\left(\Delta_i^\mu\Delta_i^\nu - \Delta_i^2 g^{\mu\nu}\right) \tag{26}$$

Doing the integration in Ω_i^* we get

$$\begin{aligned}\int d\Omega_1^*\sum_{pol}|\mathcal{M}|^2 &= (gM_W)^2\frac{1}{|D(\Delta_i^2)|^2}\left(-g^{\alpha\beta}+\frac{k^\alpha k^\beta}{M_W^2}\right)\\ &\times\left(g_{\mu\alpha}+T^W_{\mu\alpha}\right)\left(g_{\nu\beta}+T^W_{\nu\beta}\right)\\ &\times\frac{48\pi\Gamma_i}{M_W}\frac{4\pi}{3}\left(\Delta_i^\mu\Delta_i^\nu - \Delta_i^2 g^{\mu\nu}\right)\end{aligned} \tag{27}$$

If we compare (8) with (27) we can write this last equation in the form

$$\begin{aligned}\int d\Omega_1^*\sum_{pol}|\mathcal{M}|^2 &= (gM_W)^2\frac{1}{|D(\Delta_i^2)|^2}\frac{M_H^4}{4M_W^2}\frac{48\pi\Gamma_i}{M_W}\frac{4\pi}{3}\\ &\times\Big[\lambda(M_W^2,\Delta_i^2;M_H^2) + 12\frac{M_W^2\Delta_i^2}{M_H^4}\\ &+X(k,\Delta_i,M_H,T^W)\Big]\end{aligned} \tag{28}$$

We get therefore

$$\frac{d\Gamma}{d\Delta_i^2 d\Omega_{\mathbf{k}}} = \frac{(2\pi)^{-5}}{128 M_H} \sqrt{\lambda(M_W^2, \Delta_i^2; M_H^2)}\,(g M_W)^2 \times \frac{1}{|D(\Delta_i^2)|^2} \frac{M_H^4}{4M_W^2} \frac{48\pi\Gamma_i}{M_W} \frac{4\pi}{3} \Big[\lambda(M_W^2, \Delta_i^2; M_H^2) + 12\frac{M_W^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^W)\Big] \quad (29)$$

Next we realize that in (29) there is no dependence on the solid angle of the real W. We can therefore trivially perform that integration. We get

$$\frac{d\Gamma}{d\Delta_i^2} = \frac{(2\pi)^{-5}}{128 M_H} (4\pi) \sqrt{\lambda(M_W^2, \Delta_i^2; M_H^2)}\,(g M_W)^2 \times \frac{1}{|D(\Delta_i^2)|^2} \frac{M_H^4}{4M_W^2} \frac{48\pi\Gamma_i}{M_W} \frac{4\pi}{3} \Big[\lambda(M_W^2, \Delta_i^2; M_H^2) + 12\frac{M_W^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^W)\Big] \quad (30)$$

and finally we get for the width

$$\Gamma = \frac{G_F M_H^3}{8\pi\sqrt{2}} \sqrt{\lambda(M_W^2, \Delta_i^2; M_H^2)} \frac{1}{\pi} \int d\Delta_i^2 \frac{\Gamma_i M_W}{|D(\Delta_i^2)|^2} \times \Big[\lambda(M_W^2, \Delta_i^2; M_H^2) + 12\frac{M_W^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^W)\Big] \quad (31)$$

or

$$\Gamma = \frac{1}{\pi} \int d\Delta_i^2 \frac{\Gamma_i M_W}{|D(\Delta_i^2)|^2}\, \Gamma_0^W(k, \Delta_i, M_H) \quad (32)$$

where

$$\Gamma_0^W(k, \Delta_i, M_H) = \frac{G_F M_H^3}{8\pi\sqrt{2}} \sqrt{\lambda(M_W^2, \Delta_i^2; M_H^2)} \times \Big[\lambda(M_W^2, \Delta_i^2; M_H^2) + 12\frac{M_W^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^W)\Big] \quad (33)$$

If we sum over all the final states of the W we can substitute Γ_i with Γ_W. (32) is in agreement with [9] and it is also in agreement with (6) of [5] in the zero width limit. Similar considerations apply to the case of the decay $H \to Z^0 + f_i \overline{f}_i$. We can summarize the final result in the formula,

$$\Gamma = \frac{1}{\pi} \int d\Delta_i^2 \frac{\Gamma_V M_V}{|D(\Delta_i^2)|^2}\, \Gamma_0^V(k, \Delta_i, M_H) \quad (34)$$

where

$$\Gamma_0^V(k, \Delta_i, M_H) = \delta_V \frac{G_F M_H^3}{16\pi\sqrt{2}} \sqrt{\lambda(M_V^2, \Delta_i^2; M_H^2)} \times \Big[\lambda(M_V^2, \Delta_i^2; M_H^2) + 12\frac{M_V^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^V)\Big] \quad (35)$$

δ_V was defined before, X is given in (7) and $k^2 = M_V^2$.

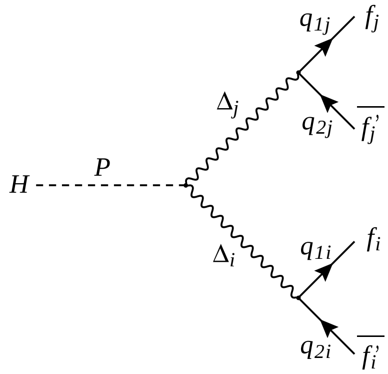


Fig. 3.

2.4 The off–shell decay $H \to V^* V^*$

We now consider the off–shell decay $H \to V^* V^*$. To be precise we derive the expression for $H \to W^{+*} W^{-*} \to (f_i \overline{f'}_i) + (f_j \overline{f'}_j)$ and then present a final result valid for all cases. We consider the kinematics given in Fig. 3, where $(f_i, \overline{f'}_i)$ represents one of the decay channels of the W^- and $(f_j, \overline{f'}_j)$ represents one of the decay channels of the W^+. After we have done the case $H \to VV^*$ it is very easy to do this case.
The expression for the width is [11],

$$d\Gamma = \frac{(2\pi)^4}{2M_H} \sum_{pol} |\mathcal{M}|^2 d\Phi_4 \quad (36)$$

where $d\Phi_4$ is the phase space for 4 particles that we write as [11]

$$d\Phi_4(P; k, q_1, q_2) = d\Phi_2(P; \Delta_i, \Delta_j) d\Phi_2(\Delta_i; q_{i1}, q_{i2})(2\pi)^3 \times d\Delta_i^2\, d\Phi_2(\Delta_j; q_{j1}, q_{j2})(2\pi)^3 d\Delta_j^2 \quad (37)$$

with

$$\Delta_i = q_{i1} + q_{i2}\ ;\ \Delta_i^2 = (q_{i1} + q_{i2})^2 \qquad \Delta_j = q_{j1} + q_{j2}\ ;\ \Delta_j^2 = (q_{j1} + q_{j2})^2 \quad (38)$$

But the 2–body phase spaces can be written as

$$d\Phi_2(\Delta_i; q_{i1}, q_{i2}) = \frac{(2\pi)^{-6}}{8}\, d\Omega_{i1}^*$$

$$d\Phi_2(\Delta_j; q_{j1}, q_{j2}) = \frac{(2\pi)^{-6}}{8}\, d\Omega_{j1}^*$$

$$d\Phi_2(P; \Delta_i, \Delta_j) = \frac{(2\pi)^{-6}}{8} \sqrt{\lambda(\Delta_i^2, \Delta_j^2; M_H^2)}\, d\Omega_{\mathbf{\Delta}_i} \quad (39)$$

where, as before, we consider that the decays products of the $W^\pm$ are massless. Putting everything together we have

$$d\Gamma = \frac{(2\pi)^{-8}}{2^{10} M_H} \sqrt{\lambda(\Delta_i^2, \Delta_j^2; M_H^2)} \times \sum_{pol} |\mathcal{M}|^2 d\Delta_i^2\, d\Delta_j^2\, d\Omega_{\mathbf{\Delta}_i}\, d\Omega_{i1}^* d\Omega_{j1}^* \quad (40)$$

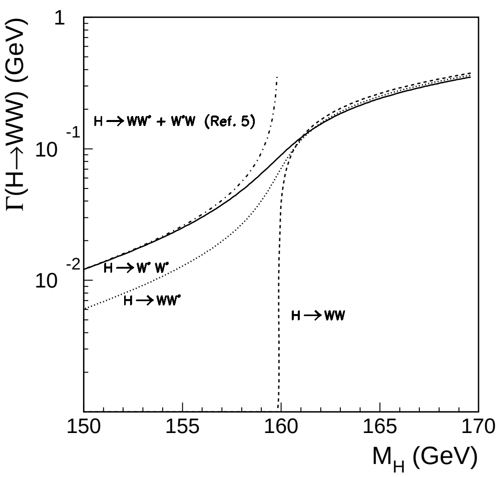


Fig. 4. Comparison of the off–shell and on–shell formulas for $H \to W^+W^-$. The dashed line corresponds to the on–shell formula (12), the dotted line to the case that only one W is off-shell (31), and the solid line corresponds to the case where both W's are off–shell (43). For comparison is also shown (6) of [5]

The matrix element is

$$\mathcal{M} = (gM_W)\,\frac{1}{D(\Delta_i^2)}\,\frac{1}{D(\Delta_j^2)}\,\frac{g}{2\sqrt{2}}\,\overline{u}(q_{i1})\gamma^\mu(1-\gamma_5)v(q_{i2}) \times\frac{g}{2\sqrt{2}}\,\overline{u}(q_{j1})\gamma^\mu(1-\gamma_5)v(q_{j2}) \qquad (41)$$

and the same procedure that we used for the $H \to VV^*$ case gives

$$\int d\Omega^*_{1i}\,d\Omega^*_{1j}\sum_{pol}|\mathcal{M}|^2 = (gM_W)^2\,\frac{1}{|D(\Delta_i^2)|^2}\,\frac{1}{|D(\Delta_j^2)|^2} \times\frac{M_H^4}{4}\,\frac{(48\pi)^2\Gamma_i\Gamma_j}{M_W^2}\left(\frac{4\pi}{3}\right)^2 \times\left[\lambda(\Delta_i^2,\Delta_j^2;M_H^2)+12\frac{\Delta_i^2\Delta_j^2}{M_H^4} + X(\Delta_i,\Delta_j,M_H,T^W)\right] \qquad (42)$$

and after doing the $d\Omega_{\mathbf{\Delta}_i}$ integration we obtain

$$\Gamma = \frac{G_F M_H^3}{8\pi\sqrt{2}}\sqrt{\lambda(\Delta_i^2,\Delta_j^2;M_H^2)}\,\frac{1}{\pi}\int d\Delta_i^2\frac{\Gamma_i M_W}{|D(\Delta_i^2)|^2}\,\frac{1}{\pi} \times\int d\Delta_j^2\frac{\Gamma_j M_W}{|D(\Delta_j^2)|^2}\left[\lambda(\Delta_i^2,\Delta_j^2;M_H^2)+12\frac{\Delta_i^2\Delta_i^2}{M_H^4} + X(\Delta_i,\Delta_j,M_H,T^W)\right] \qquad (43)$$

Summing over all final states we get

$$\Gamma = \frac{1}{\pi}\int d\Delta_i^2\frac{\Gamma_V M_V}{|D(\Delta_i^2)|^2}\,\frac{1}{\pi} \times\int d\Delta_j^2\frac{\Gamma_V M_V}{|D(\Delta_j^2)|^2}\,\Gamma_0^V(\Delta_i,\Delta_j,M_H) \qquad (44)$$

where[1]

$$\Gamma_0^V(\Delta_i,\Delta_j,M_H) = \delta_V\,\frac{G_F M_H^3}{16\pi\sqrt{2}}\,\sqrt{\lambda(\Delta_i^2,\Delta_j^2;M_H^2)} \times\left[\lambda(\Delta_i^2,\Delta_j^2;M_H^2)+12\frac{\Delta_i^2\Delta_j^2}{M_H^4} + X(\Delta_i,\Delta_j,M_H,T^V)\right] \qquad (46)$$

This result is in agreement with [10], except for the value of δ_Z. One should mention that formulas for off–shell decays of the type of (34) and (44) for other decays are known in the literature [16].

2.5 A comparison of the various formulas

Perhaps it is useful to indicate the domain of validity of the various formulas for the widths. This will depend on the value of the Higgs boson mass. In Fig. 4 we plot the various formulas for the case of $H \to W^+W^-$.

From this figure it is clear that the proper way to calculate the width below the two W's threshold is to use (43) with the two W's off–shell. The two integrations in (43) automatically take care of the fact that either one of the W's can be close to be on–shell. In (6) of [5] this is done by adding the two possibilities, but as the width is neglected the formula is only good below the threshold.

[1] One might worry about the factor δ_V. For the $W^{+*}W^{-*}$ final case there is no problem because the final states of the W^{+*} are different from the final states of the W^{-*}. Therefore

$$\sum_{Final\ States}\Gamma\left[H\to(W^{+*}\to i_+)+(W^{-*}\to i_-)\right]$$

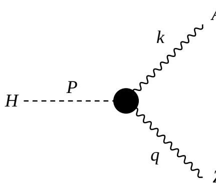
$$\propto\sum_{i_+}\sum_{i_-}\Gamma(W^{+*}\to i_+)\Gamma(W^{-*}\to i_-)$$

$$=\sum_{i_+}\Gamma(W^{+*}\to i_+)\sum_{i_-}\Gamma(W^{-*}\to i_-)=\Gamma_W\Gamma_W \qquad (45)$$

and $\delta_W = 2$ because of the constants we factored out. For the $H \to (Z^* \to i) + (Z^* \to j)$ case one should be more careful. If $i \neq j$ than we should divide by 2 otherwise we would be double counting in the product $(\Gamma_1+\Gamma_2+\cdots)(\Gamma_1+\Gamma_2+\cdots)$. For $i = j$ there is no double counting in the above product, but now we have two pairs of identical particles in the final state but we also have 2 diagrams. Then we should square the sum of the amplitudes and divide by 4. In general this would not give a factor of 1/2 because of the interference term. However the interference will be negligible because the momenta squared in the denominators cannot be equal to M_Z in all 4 lines (of the product of the 2 diagrams) at the same time. Therefore if we neglect the interference we should divide also by 2 in this case. Therefore $\delta_Z = 1$

3 The decay $H \to \gamma Z$

Due to the electromagnetic gauge invariance, the most general expression for the coupling $H\gamma Z$ for the case of on–shell γ and Z is,



$$-i\,\frac{e^2 g}{16\pi^2 M_W}(g_{\mu\nu} k\cdot q - k_\nu q_\mu) \times A(q^2 = M_Z^2, M_H) \tag{47}$$

Fig. 5.

where $A(q^2, M_H)$ is a dimensionless form factor that depends only in the mass of the H and on the square of the momentum of the Z (if the Z is on–shell then $q^2 = M_Z^2$). In the SM the lowest contribution to A is at the 1–loop level. If we are considering physics Beyond the Standard Model (BSM) then we should have

$$A = A_{SM} + A_{BSM} \tag{48}$$

where the SM contribution is given [6,12–14] by

$$A_{SM} = A_W + A_F \tag{49}$$

with[2]

$$\begin{aligned} A_W = -4\cot\theta_W \Bigg[&(3 - \tan\theta_W^2)\ J_1(q^2, M_H^2, M_W^2) \\ &+ \left(-5 + \tan_W^2\theta_W - \frac{1}{2}\frac{M_H^2}{M_W^2}(1 - \tan\theta_W^2)\right) \\ &\times J_2(q^2, M_H^2, M_W^2)\Bigg] \end{aligned} \tag{50}$$

and

$$\begin{aligned} A_F = -\sum_f &\frac{4 g_V^f Q_f}{\sin\theta_W \cos\theta_W} \\ &\times \left[-J_1(q^2, M_H^2, M_f^2) + 4 J_2(q^2, M_H^2, M_f^2)\right] \end{aligned} \tag{51}$$

where Q_f is the charge, in units of $|e|$, of the fermion f in the loop, and $g_V^f = 1/2 T_3^f - Q_f \sin^2\theta_W$. The explicit form of the functions J_1 and J_2 in the 't Hooft-Feynman gauge can be found in Appendix B. In the following we will use this general coupling to evaluate both the on–shell and the off–shell decays of the Higgs boson. The case of the off–shell decays needs some further discussion. In fact it was shown in [12] that the off–shell 3–point function $H \to \gamma Z^*$ is not gauge invariant. This means that there are contributions that are not of the form of (47). However it has also been shown in [12] that close to the pole of the vector boson propagator the contribution, in the 't Hooft–Feynman gauge, of the box diagrams (needed to make the whole process gauge invariant) was only of the order of $\leq 1\%$ of the gauge invariant part in (47). Although we have not done here the full calculation, we are making the assumption that close to the Z resonance (which gives the dominant contribution to $H \to \gamma Z^* \to \gamma f\overline{f}$) the same result applies. We also do not think that a full calculation is here necessary because the SM contribution from $H \to \gamma Z$ to the total Higgs boson width is very small as it will be shown in Sect. 6.

[2] Our convention here for the coupling, (47) is as in [6]. It differs from our previous convention, [13], by a factor of $-1/\sin\theta_W$. Our conventions are explained in Appendix A

3.1 The on–shell decay $H \to \gamma Z$

The differential width is, like before (see (2))

$$d\Gamma = \frac{1}{32\pi^2} \sum_{pol} |\mathcal{M}|^2 \frac{|\mathbf{k}|}{M_H^2}\, d\Omega_{\mathbf{k}} \tag{52}$$

where

$$\mathcal{M} = \epsilon^\mu(k)\epsilon^\nu(q)\,\frac{e^2 g}{16\pi^2 M_W}\ (g_{\mu\nu}\, k\cdot q - k_\nu q_\mu)\, A(q^2, M_H) \tag{53}$$

We get therefore

$$\sum_{pol} |\mathcal{M}|^2 = \left(\frac{e^2 g}{16\pi^2 M_W}\right)^2 2(k\cdot q)^2\, |A|^2 \tag{54}$$

Now using

$$|\mathbf{k}| = \frac{k\cdot q}{M_H} = \frac{1}{2}\, M_H \sqrt{\lambda(M_Z^2, 0; M_H^2)} \tag{55}$$

where

$$\lambda(M_Z^2, 0; M_H^2) = \left(1 - \frac{M_Z^2}{M_H^2}\right)^2 \tag{56}$$

we get finally

$$\Gamma = \frac{G_F M_H^3}{4\pi\sqrt{2}}\, \frac{\alpha^2}{16\pi^2}\, \lambda(M_Z^2, 0; M_H^2)^{3/2}\, |A|^2 \tag{57}$$

This result is in agreement for the SM with [6,12] but it differs by a factor of two from [8] that claims to have the same definition of A as we and [6] do.

3.2 The off–shell decay $H \to \gamma Z^*$

We consider for definiteness the decay $H \to \gamma f_i \overline{f}_i$. According to the discussion above we use the off–shell coupling given by (47) which is a good approximation for q^2 close to M_Z^2, which is the dominant contribution to $H \to \gamma f_i \overline{f}_i$. This is represented in Fig. 6.
The differential width can be written as in (18)

$$d\Gamma = \frac{(2\pi)^{-5}}{128 M_H} \sqrt{\lambda(0, \Delta^2; M_H^2)} \sum_{pol} |\mathcal{M}|^2\, d\Omega_{\mathbf{k}}\, d\Delta^2 d\Omega_1^* \tag{58}$$

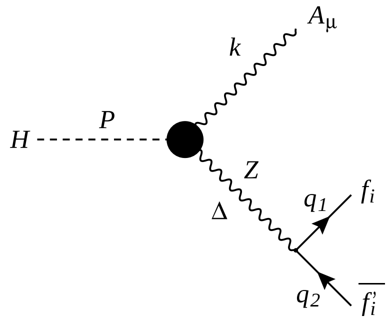


Fig. 6.

where the matrix element $\mathcal{M}$ is (we again neglect the fermion masses)

$$\mathcal{M} = \epsilon^\mu(k) \frac{e^2 g}{16\pi^2 M_W} \left(g_{\mu\nu}\, k\cdot\Delta - k_\nu \Delta_\mu\right) A(\Delta^2, M_H) \\ \frac{1}{D(\Delta^2)} \frac{g}{\cos\theta_W} \overline{u}(q_1)\gamma^\nu (g_V^f - g_A^f \gamma_5) v(q_2) \tag{59}$$

and

$$\Delta = q_1 + q_2 \tag{60}$$

Our conventions for the couplings of the Z to the fermion f are given in Appendix A. The sum over polarizations and spins of the matrix element squared gives now

$$\sum_{pol} |\mathcal{M}|^2 = \left(\frac{e^2 g}{16\pi^2 M_W)}\right)^2 \frac{1}{|D(\Delta^2)|^2} \left(\frac{g}{\cos\theta}\right)^2 8|A|^2 \\ \times \left(g_V^{f\,2} + g_A^{f\,2}\right) \Big[k\cdot\Delta\; k\cdot q_1\; \Delta\cdot q_2 \\ + k\cdot\Delta\; k\cdot q_2\; \Delta\cdot q_1 - k\cdot q_1\; k\cdot q_2\; \Delta\cdot\Delta \Big] \tag{61}$$

Using now (25) to perform the integration over the solid angle in the center of mass frame of the decaying Z we get

$$\int d\Omega_1^* \sum_{pol} |\mathcal{M}|^2 = \left(\frac{e^2 g}{16\pi^2 M_W}\right)^2 \frac{1}{|D(\Delta^2)|^2} \left(\frac{g}{\cos\theta}\right)^2 |A|^2 \\ \times \left(g_V^{f\,2} + g_A^{f\,2}\right) \frac{32\pi}{3} (k\cdot\Delta)^2\, \Delta^2 \tag{62}$$

We can now perform the integration over the solid angle of the photon and obtain

$$\frac{d\Gamma}{d\Delta^2} = \frac{1}{32\pi^2 M_H} \lambda(\Delta^2, 0; M_H^2)^{3/2} M_H^3 \\ \left(\frac{e g^2}{16\pi^2 M_W}\right)^2 \frac{\Gamma_i}{M_Z} \Delta^2 \frac{1}{|D(\Delta^2)|^2} \tag{63}$$

where we have used the expression for the partial width Γ_i of $Z \to f_i \overline{f}_i$

$$\Gamma_i = \frac{1}{12\pi} \left(\frac{g}{\cos\theta_W}\right)^2 \left(g_V^{f\,2} + g_A^{f\,2}\right) \tag{64}$$

Summing over all the final states we obtain finally

$$\Gamma = \frac{1}{\pi} \int d\Delta^2\, \frac{\Gamma_Z}{M_Z} \frac{\Delta^2}{|D(\Delta^2)|^2}\, \Gamma^{\gamma Z}(M_H, \Delta^2) \tag{65}$$

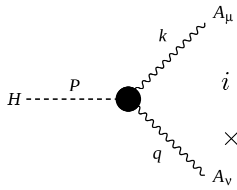
where

$$\Gamma^{\gamma Z}(M_H, \Delta^2) = \frac{G_F M_H^3}{4\pi\sqrt{2}} \frac{\alpha^2}{16\pi^2} \\ \lambda(\Delta^2, 0; M_H^2)^{3/2} |A(\Delta^2, M_H^2)|^2 \tag{66}$$

is the decay into an off-shell Z and gives back (57) when $\Delta^2 = M_Z^2$.

4 The decay $H \to \gamma\gamma$

For completeness we also give the general formula for this decay. Due to the electromagnetic gauge invariance the most general expression for the coupling[3] $H\gamma\gamma$ is



$$i \frac{e^2 g}{16\pi^2 M_W} \left(g_{\mu\nu}\, k\cdot q - k_\nu q_\mu\right) \\ \times I(q^2 = 0, M_H) \tag{67}$$

Fig. 7.

where, as before,

$$I = I_{SM} + I_{BSM} \tag{68}$$

The Standard Model contribution is given by [8,6,12]

$$I_{SM} = I_W + I_F \tag{69}$$

where

$$I_W = -4 \left[-4 J_1(q^2, M_H^2, M_W^2) \\ + \left(6 + \frac{M_H^2}{M_W^2}\right) J_2(q^2, M_H^2, M_W^2) \right] \tag{70}$$

and

$$I_F = \sum_f 4 Q_f^2 \left[-J_1(q^2, M_H^2, M_f^2) + 4 J_2(q^2, M_H^2, M_f^2) \right]. \tag{71}$$

Using the above coupling and comparing with the case $H \to \gamma Z$, (52), (53) e (54), it is straightforward to obtain

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^3 M_H^3}{256\pi^2 \sin^2\theta_W M_W^2} |I|^2 \tag{72}$$

in agreement with [6,8,14].

[3] For on–shell photons

5 The 3–point functions $Z^* \to H\gamma$ and $\gamma^* \to H\gamma$

For some applications it is also important to know the related off–shell 3-point functions $Z^* \to H\gamma$ and $\gamma^* \to H\gamma$. For completeness we collect them here. We should however warn the reader that we only include here the gauge invariant part of the one loop diagrams contributing to those functions and therefore the limit of applicability of the expressions below is restricted to the case where that is the dominant contribution. In the other cases more diagrams have to be considered to render the physical amplitudes gauge invariant. General expressions for the off–shell 3–point functions can be found in [12,14].

5.1 The $Z^* \to H\gamma$ 3–point function

We use the results of [12,13]. The amplitude can be written as[4]

$$i\,\mathcal{M} = i\,\epsilon_Z^\nu(q)\,\epsilon_A^\mu(k)\left(\frac{e^2 g}{16\pi^2 M_W}\right)(g_{\mu\nu}\,k\cdot q - k_\nu q_\mu)\;\; A(q^2, M_H) \tag{73}$$

where

$$A = A_{SM} + A_{BSM} \tag{74}$$

The standard model dimensionless amplitude A_{SM} is given by (49). The sign difference between (49) and (73) is due to the fact that in the first one q is an *outgoing* momentum and in the last one is *incoming*.

5.2 The $\gamma^* \to H\gamma$ 3–point function

Again using [12] we have

$$i\,\mathcal{M} = -i\,\epsilon_A^\nu(q)\,\epsilon_A^\mu(k)\left(\frac{e^2 g}{16\pi^2 M_W}\right)(g_{\mu\nu}\,k\cdot q - k_\nu q_\mu)\;\; I(q^2, M_H) \tag{75}$$

where

$$I = I_{SM} + I_{BSM} \tag{76}$$

The standard model dimensionless amplitude I_{SM} is given by (69).

5.3 An effective Lagrangian for the SM couplings

We can write an effective Lagrangian that reproduces the couplings given in (47), (67), (73), (75). This is specially useful if we want to add new physics, in addition to the SM, as we will show in the next section. We get

$$\mathcal{L}^{eff} = -\frac{1}{4}\,A_{\mu\nu}A^{\mu\nu}H\;\mathcal{I}_{SM} + \frac{1}{2}\,A_{\mu\nu}Z^{\mu\nu}H\;\mathcal{A}_{SM} \tag{77}$$

[4] Notice that our conventions here differ by a factor $-1/\sin\theta_W$ with respect to [13]

where we have defined

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad ; \quad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \tag{78}$$

and

$$\begin{aligned} \mathcal{I}_{SM} &= \frac{e^2 g}{16\pi^2 M_W}\; I_{SM}(q^2, M_H) \\ \mathcal{A}_{SM} &= \frac{e^2 g}{16\pi^2 M_W}\; A_{SM}(q^2, M_H) \end{aligned} \tag{79}$$

The effective Lagrangian, (77), is valid for the case of one on–shell photons and Z^0's. A_{SM} and I_{SM} are given in (49) and (69).

6 An example of extension of the SM

A possible enhancement of the production and decay rates of the Higgs boson can be originated by an anomalous couplings of the Higgs boson to the vector bosons. These interactions can be described in terms of an effective dimension-six term in the interaction Lagrangian density

$$\mathcal{L}_{eff} = \sum_{i=1}^{7} \frac{f_i}{\Lambda^2} O_i \tag{80}$$

where the O_i are the operators which represent the anomalous couplings, Λ is the typical energy scale of the interaction and f_i are the constants which define the strength of each term [7,8].

The anomalous couplings $H\gamma\gamma$, HZZ, $HZ\gamma$ and HWW follow from the effective Lagrangian (80) and can be written in the unitary gauge [7,8] as,

$$\begin{aligned} \mathcal{L}_{eff}^{HVV} = g\frac{m_W}{\Lambda^2}\Bigg[&-\frac{s^2(f_{BB}+f_{WW}-f_{BW})}{2} H A_{\mu\nu}A^{\mu\nu} \\ &+ \frac{2m_W^2}{g^2}\frac{f_{\phi,1}}{c^2} H Z_\mu Z^\mu + \frac{c^2 f_W + s^2 f_B}{2c^2} Z_{\mu\nu}Z^\mu(\partial^\nu H) \\ &- \frac{s^4 f_{BB} + c^4 f_{WW} + s^2c^2 f_{BW}}{2c^2} H Z_{\mu\nu}Z^{\mu\nu} \\ &+ \frac{s(f_W - f_B)}{2c} A_{\mu\nu}Z^\mu(\partial^\nu H) \\ &+ \frac{s(2s^2 f_{BB} - 2c^2 f_{WW} + (c^2-s^2) f_{BW})}{2c} H A_{\mu\nu}Z^{\mu\nu} \\ &+ \frac{f_W}{2}(W^+_{\mu\nu}W^{-\mu} + W^-_{\mu\nu}W^{+\mu})(\partial^\nu H) \\ &- f_{WW} H W^+_{\mu\nu}W^{-\mu\nu}\Bigg] \end{aligned} \tag{81}$$

where $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ with $X = A, Z, W$, and $s(c) = \sin\theta_W(\cos\theta_W)$, respectively.

Both $f_{\phi,1}$ and f_{BW} are already severely constrained by precise measurements at low energy experiments, once

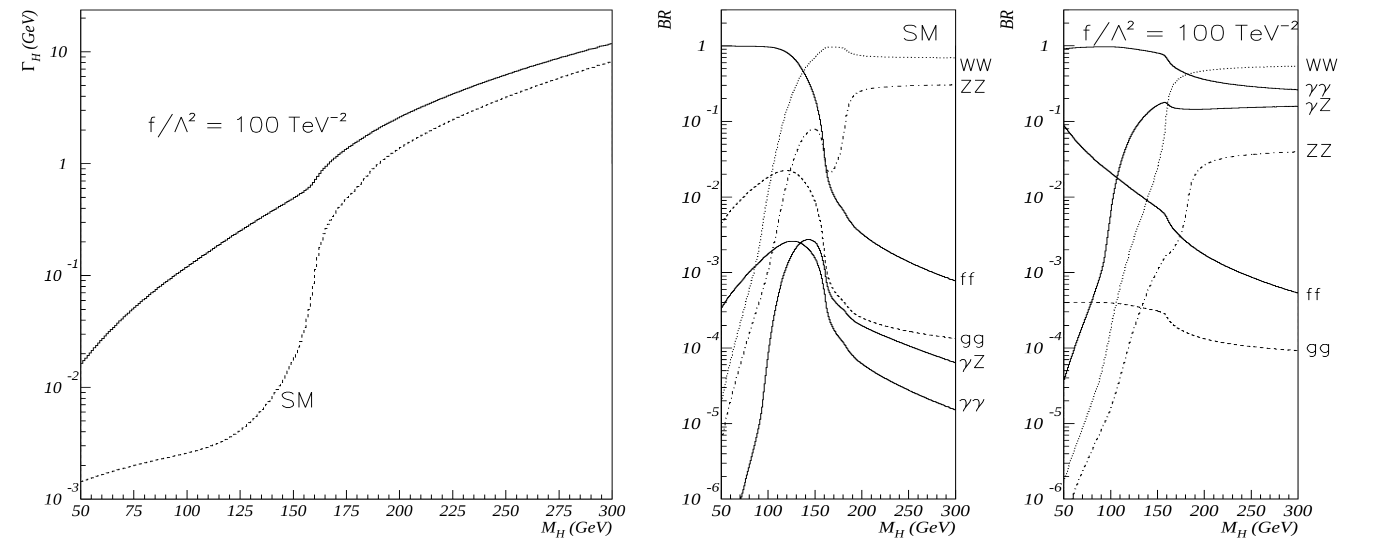


Fig. 8. Higgs Width and Branching Ratios as a function of its mass

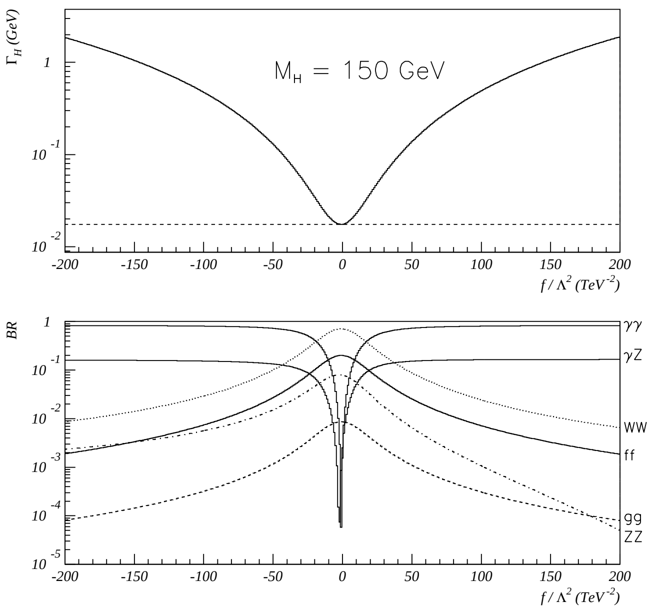


Fig. 9. Higgs width as a function of f/Λ^2

they contribute to the Z^0 mass and to the $B - W^3$ mixing, respectively. In what follows these parameters will be assumed to be zero. Under this assumption, both HWW and HZZ have the same tensorial structure. With the convention $H(p_H) \to V^\mu(p_1) + V^\nu(p_2)$ for the momenta, we have:

$$T_V^{\mu\nu} \equiv -A_V \left[p_1^\nu p_H^\mu - (p_1.p_H)g^{\nu\mu} + p_H^\nu p_2^\mu - (p_2.p_H)g^{\nu\mu}\right] + B_V \left[p_1^\nu p_2^\mu - (p_1.p_2)g^{\nu\mu}\right] \quad (82)$$

(V = Z, W), where :

$$A_W \equiv -\frac{1}{2}\frac{f_W}{\Lambda^2}$$

$$B_W \equiv -2\frac{f_{WW}}{\Lambda^2}$$

$$A_Z \equiv -\frac{1}{2}\left(\frac{f_B}{\Lambda^2}\sin^2\theta_W + \frac{f_W}{\Lambda^2}\cos^2\theta_W\right)$$

$$B_Z \equiv -2\left(\frac{f_{BB}}{\Lambda^2}\sin^4\theta_W + \frac{f_{WW}}{\Lambda^2}\cos^4\theta_W\right) \quad (83)$$

The value of $X_V(p_1, p_2, M_H, T^V)$ as defined in (7) is, thus, given by:

$$\begin{aligned}X_V = 4\Bigg\{ & A_V\left[4\frac{p_1^2p_2^2}{M_H^2} - \frac{p_1.p_2}{M_H^4}((p_1^2-p_2^2)^2 - (p_1^2+p_2^2)M_H^2)\right] \\ & + B_V\left[-6\frac{(p_1.p_2)p_1^2p_2^2}{M_H^4}\right] \\ & + A_V^2\Bigg[p_1^2p_2^2 + \frac{(p_1^2+p_2^2)(4p_1^2p_2^2 - (M_H^2-(p_1^2+p_2^2))^2)}{4M_H^2} \\ & + \frac{(M_H^4-(p_1^2-p_2^2)^2)}{4M_H^4} \\ & \times\ (4p_1^2p_2^2 + M_H^2(p_1^2+p_2^2) - (p_1^2+p_2^2)^2)\Bigg] \\ & + A_VB_V\Bigg[-2\frac{p_1^2p_2^2(M_H^2-(p_1^2+p_2^2))}{M_H^2} \\ & + \frac{p_1^2p_2^2((p_1^2-p_2^2)^2 - M_H^2(p_1^2+p_2^2))}{M_H^4}\Bigg] \\ & + B_V^2\left[\frac{p_1^2p_2^2}{2M_H^4}((M_H^2-(p_1^2+p_2^2))^2 + 2p_1^2p_2^2)\right]\Bigg\} \quad (84)\end{aligned}$$

This expression can then be used in (12), (31) and (44) to evaluate the decay widths. We have verified that if we use (84) with the definitions of (83) into (12) for the decay into two real vector bosons we recover the results of [8].

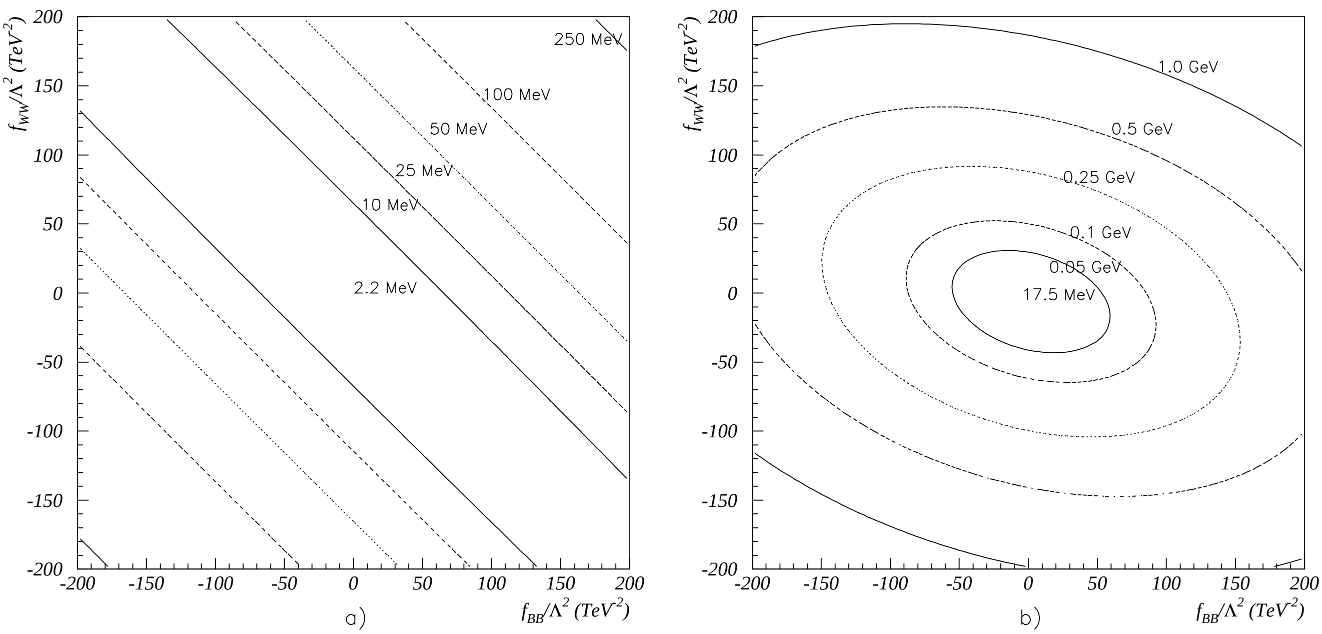


Fig. 10. Constant Higgs' Width lines as a function of f_{BB}/Λ^2 and f_{WW}/Λ^2 for: **a** $M_H = 85$ GeV and **b** $M_H = 150$ GeV

However our expressions extend those results for the off-shell case.

The decays $H\to\gamma Z$ and $H\to\gamma\gamma$ appear at tree-level, the corresponding form factors, (47) and (67), are:

$$A_{BSM} \equiv \frac{2\pi M_W^2 \tan\theta_W}{\alpha}\left[\frac{f_W}{\Lambda^2} - \frac{f_B}{\Lambda^2} + 4\left(\frac{f_{BB}}{\Lambda^2}\sin\theta_W^2 - \frac{f_{WW}}{\Lambda^2}\cos\theta_W^2\right)\right] \quad (85)$$

$$I_{BSM} \equiv \frac{8\pi M_W^2 \sin\theta_W^2}{\alpha}\left(\frac{f_{BB}}{\Lambda^2} + \frac{f_{WW}}{\Lambda^2}\right) \quad (86)$$

With these variables it is possible to compute the various Higgs decay widths, including the interference of the new terms with the Standard Model, and allowing for decays to virtual gauge bosons.

In this model the Branching Ratios to $\gamma\gamma$ and γZ increase and these decays may become dominant for some region of parameters. For $H\to WW$ and $H\to ZZ$ the new contributions may interfere constructively or destructively with the Standard Model terms. In Fig. (8) the width and branching ratios of the Higgs as a function of its mass are displayed for the Standard Model and with the new contributions where all the non-zero f_i are assumed equal and $f_i/\Lambda^2 = 100$ TeV^{-2}.

The variation of the total width and branching ratios with f/Λ^2 is shown in Fig. 9, for a Higgs boson mass of 150 GeV. In Fig. (10) all f_i except the ones contributing directly to the H decay to $\gamma\gamma$ are set to 0. The variation with f_{BB}/Λ^2 and f_{WW}/Λ^2 is displayed for two different masses: 85 GeV and 150 GeV.

7 Discussion

In this paper we derive the complete set of formulas for the decay widths of the Higgs boson in vector bosons. The formulas are valid both for the Standard Model (SM) and for any arbitrary extension. For the case of the decay into the $W^\pm$ and Z^0 the formulas are also valid for off–shell decays. This is important for Higgs boson masses close to the threshold of the production of one or two real vector bosons. As many of these results have appeared before [5] in the literature [5–10,14], sometimes for particular cases, we will now comment on the comparison of our results with those.

For the on–shell decay $H \to VV$ our final expression (12), is in agreement with [9,10]. Our final expression, (34), for the off-shell decay $H \to VV^*$ also agrees with [9, 10]. We are also in agreement with (6) of [5] in the zero width limit. For the off–shell decay $H \to V^*V^*$ our result, (44), is in agreement with [10] except for the factor δ_Z. For the on–shell decay $H \to \gamma\gamma$ we are in agreement with [6, 8,14], while for the on–shell decay $H \to \gamma Z$ we agree with [6,14] but have a factor of two difference with respect to [8]. The formulas for the off-shell decay $H \to \gamma Z^*$ are in agreement with [12,13].

As our main contribution is to extend the formulas for an arbitrary extension of the SM, including off-shell decays we studied, as an example, the case of the gauge–invariant effective Lagrangian models of [7,8]. Our expressions reproduce the results of [8] for the on–shell decays and extend them for the region of the Higgs boson mass

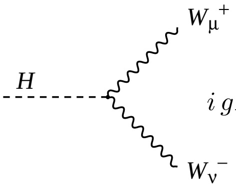
[5] After the completion of this work we became aware of a related work [19] for the Standard Model. There the vector bosons are also considered to be off-shell and the results are in agreement with ours for the particular case of the Standard Model

close to the two W's threshold where the off–shell decays have to be considered. This region is important for the studies done at the Tevatron and at LEPII where these models have been considered [17,18].

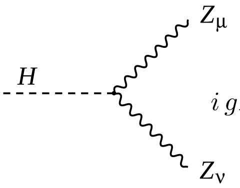
Acknowledgements. This work was supported in part by the TMR network grant ERBFMRXCT-960090 of the European Union.

Appendix A
Standard Model Feynman rules

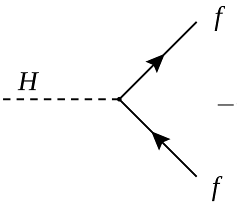
Because of the interference terms between the Standard Model (SM) and possible extensions, it is important that we state our conventions for the SM. We follow the conventions of [6]. These differ in some signs from the conventions used in [12,13]. For the convenience of the reader we collect the most important Feynman rules here.



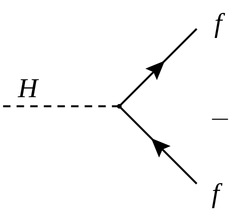
$$i\,gM_W\,g_{\mu\nu}$$



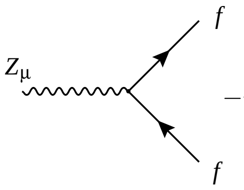
$$i\,gM_Z\,g_{\mu\nu} \tag{87}$$



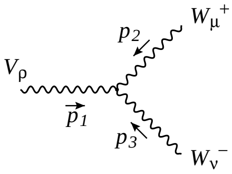
$$-i\,\frac{g\,m_f}{2M_W}$$



$$-i\,e\,Q_f\gamma^\mu \tag{88}$$



$$-i\,\frac{g}{\cos\theta_W}\gamma^\mu\left(g_V^f - g_A^f\gamma_5\right) \tag{89}$$



$$i\,g_V\left[g^{\mu\nu}(p_2-p_3)^\rho + g^{\nu\rho}(p_3-p_1)^\mu + g^{\rho\mu}(p_1-p_2)^\nu\right] \tag{90}$$

for $V = A, Z$ with $g_A = e$, $g_Z = g\cos\theta_W$ and

$$g_V^f = \frac{1}{2}\,T_3^f - Q_f\sin^2\theta_W \quad ; \quad g_A^f = \frac{1}{2}\,T_3^f \tag{91}$$

where Q_f is the charge of fermion f in units of $|e|$.

Appendix B
The J_1 and J_2 functions

The explicit expressions for the functions J_1 and J_2 introduced in Sect. 3, are [12,13]

$$\begin{aligned} J_1(q^2, M_H^2, M_X^2) &= -M_W^2\ C_0(q^2, 0, M_H^2, M_X^2, M_X^2, M_X^2) \\ J_2(q^2, M_H^2, M_X^2) &= \frac{1}{2}\frac{M_X^2}{q^2 - M_H^2}\left[1 + 2M_X^2 \right. \\ &\quad \times C_0(q^2, 0, M_H^2, M_X^2, M_X^2, M_X^2) \\ &\quad + \frac{q^2}{q^2 - M_H^2}\left(B_0(q^2, M_X^2, M_X^2) \right. \\ &\quad \left.\left. -\ B_0(M_H^2, M_X^2, M_X^2)\right)\right] \end{aligned} \tag{92}$$

where B_0 and C_0 are the Passarino-Veltman functions[15] and M_X is the mass of the particle in the loop. These functions were calculated in [12,13] in the 't Hooft-Feynman gauge. They are are related to the functions I_1 and I_2 of [6] by the following relations

$$\begin{aligned} J_1(q^2, M_H^2, M_X^2) &= I_2(\tau_X, \lambda_X) \\ J_2(q^2, M_H^2, M_X^2) &= \frac{1}{4}\ I_1(\tau_X, \lambda_X) \end{aligned} \tag{93}$$

with

$$\tau_X = \frac{4M_X^2}{M_H^2} \quad ; \quad \lambda_X = \frac{4M_X^2}{q^2} \tag{94}$$

With these relations it is easy to verify that (49) is in agreement with (2.22) of [6]. To verify the equivalence of (69) with (2.16) and (2.17) of [6] for the on–shell decay $H \to \gamma\gamma$ one has to note that

$$J_1(0, M_H^2, M_X) = I_2(\tau_X, \infty) = \frac{\tau_X}{2}\ f(\tau_X)$$

$$J_2(0, M_H^2, M_X) = \frac{1}{4}\ I_1(\tau_X, \infty)$$

$$= -\frac{\tau_X}{8} + \frac{\tau_X^2}{8}\, f(\tau_X) \qquad (95)$$

where $f(\tau)$ is defined in (2.19) of [6]. Then we get for the W contribution

$$\begin{aligned} I_W &= -4\left[-4J_1(0, M_H^2, M_W^2) \right.\\ &\quad \left. + \left(6 + \frac{M_H^2}{M_W^2}\right) J_2(0, M_H^2, M_W^2)\right] \\ &= 16\, J_1(0, M_H^2, M_W^2) - \left(24 + \frac{16}{\tau_W}\right) J_2(0, M_H^2, M_W^2) \\ &= 2 + 3\tau_W + 3\tau_W(2 - \tau_W)\, f(\tau_W) \qquad (96) \end{aligned}$$

and for a fermion of charge Q_f

$$\begin{aligned} I_F &= 4\, Q_f^2 \left[- J_1(0, M_H^2, M_W^2) + 4J_2(0, M_H^2, M_W^2)\right] \\ &= Q_f^2 \left[-2\,\tau_f f(\tau_f) - 2\,\tau_f + 2\tau_f^2 f(\tau_f)\right] \\ &= Q_f^2 \left[-2\,\tau_f \left(1 + (1 - \tau_f)\, f(\tau_f)\right)\right] \qquad (97) \end{aligned}$$

in agreement with (2.17) of [6].

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